Weakly dissipative dust-ion-acoustic solitons

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We investigate the possibility for dust ion-acoustic solitons to exist. Compressive solitonlike perturbations are damped and slowed down, mainly due to the plasma absorption and ion scattering on microparticles. The perturbations are shown to possess the main properties of solitons. There is a principal possibility to study experimentally the role of trapped electrons in the soliton formation.

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I. INTRODUCTION

A complex (dusty) plasma is the plasma containing electrons, ions, neutrals, and solid or liquid (dust) microparticles. The remarkable property of complex plasmas is the particle charging process [1]. Usually, in laboratory experiments the microparticles are negatively charged, due to the electron and ion fluxes on the particle surface. Any fluctuations in plasma parameters can vary these fluxes and thus cause fluctuations of the microparticle charge.

Nonlinear coherent and dissipative structures in complex plasmas can be formed by different means. These are not necessarily restricted to the mode excitation due to instabilities, or an external forcing, but can also be a regular collective process analogous to the shock wave generation in gas dynamics. The *anomalous* dissipation in complex plasmas, which originates from the dust particle charging process, makes possible existence of a new kind of shocks related to this dissipation [2,3]. In the absence of dissipation (or if the dissipation is weak at the characteristic dynamical time scales of the system) the balance between nonlinear and dispersion effects can result in the formation of a symmetrical solitary wave—a soliton. Investigation of the anomalous dissipation is especially interesting at the ion-acoustic time scales, when "massive" microparticles can be treated as motionless. The charging processes at these time scales are usually not in equilibrium and, hence, the role of anomalous dissipation might be crucial [2,4]. So far, study of nonlinear structures at ion-acoustic time scales (in complex plasmas) was mostly related to shocks [2,3,5,6]. There has also been an experimental investigation of dust-ion-acoustic (DIA) solitons [7]. The first theoretical study of DIA solitons in complex plasmas [8] used an approximation neglecting absorption and scattering of electrons and ions by microparticles. These processes, resulting in the anomalous dissipation, make the existence of "pure" steady-state nonlinear structures impossible. For DIA shocks, this dissipation was shown to cause qualitatively new effects [2,3]. The influence of the anomalous dissipation on DIA solitons is still an open question.

The purpose of this paper is to determine and to investigate the situation where the compressive DIA solitonlike perturbations can exist in complex plasmas and to give a treatment of these perturbations. In Sec. II we present the model that describes nonlinear DIA perturbations and show the importance of electrons trapped by the soliton in complex plasmas. In Sec. III we define a "weakly dissipative" regime for DIA solitonlike perturbations, study time evolution of the individual DIA soliton, and investigate the interaction of two weakly dissipative solitons. A summary of our findings is given in Sec. IV.

II. MODEL

We use the model [3,9] based on a set of fluid equations (which take into account the variation of ion density and the ion momentum dissipation due to interaction with microparticles), Poisson equation, charging equation for microparticles, and include the ionization process. We assume that the following approximations are valid: plasma can be considered as uniform and unmagnetized; size of particles is much smaller than the electron Debye length and the distance between microparticles; microparticles can be considered as stationary, so that their density n_d is constant in the ion-acoustic time scale [8]; electron and ion temperatures are assumed to be constant, and their ratio T_e/T_i is sufficiently large (the latter allows us to neglect the Landau damping for ions [10,11]); charge variation on microparticles is solely due to variation of the plasma potential, and the charging is described by the orbit-motion-limited (OML) model [1,12,13].

We do not take into account any heat transfer processes that might influence the propagation and evolution of ion-acoustic perturbation—ion-neutral and electron-neutral collisions are neglected: Under the experimental conditions of Ref. [6] (neutral argon gas pressure is $\sim 10^{-4}$ torr and the electron temperature is $T_e = 1.5$ eV), the electron mean free path exceeds 10^4 cm and the ion mean free path is of the order of 10^3 cm. These scales are much larger than the scales of the device (90 cm length and 40 cm diameter, Ref. [14]). In another experiment [5], the neutral gas pressure was below 10^{-5} torr and therefore the collisionless approximation is completely justified. Thus, the only important dissipation in the system is related to the plasma absorption on microparticles as well as the ion scattering on microparticles [2,3].

The evolution equations for the ion density n_i and the ion drift velocity u are

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$$\frac{\partial n_i}{\partial t}+\frac{\partial(n_i u)}{\partial x}=-\nu_{\rm r} n_i+\nu_{\rm r0} n_{i0}, \qquad (1)$$

$$\frac{\partial(n_i u)}{\partial t}+\frac{\partial(n_i u^2)}{\partial x}+\frac{e n_i}{m_i}\frac{\partial \varphi}{\partial x}=-\nu_{id} n_i u. \qquad (2)$$

Here φ is the electrostatic plasma potential, $\nu_{\rm r}$ is the frequency of ion recombination on microparticles, and ν_{id} is the momentum-transfer frequency due to ion-particle collisions. The ionization rate is $\nu_{\rm r0} n_{i0}$ (subscript 0 denotes unperturbed variables). The latter is chosen to be independent of the electron density and can be considered to be constant. This assumption is valid [3] under the conditions of the experiments on nonlinear wave excitation carried out on a Q machine [5] and a double plasma device [6,7]. Indeed, in the laboratory experiments of Ref. [5], a hot ($\sim$2000–2500 K) plate installed in the end region of the machine was irradiated with a beam of cesium atoms, so that cesium ions in the plasma were produced through ionization of cesium atoms at the plate surface. In the experiments of Refs. [6,7], the electron mean free paths were so long that the neutrals were ionized presumably in collisions with the wall. Thus, under the experimental conditions of Refs. [6,7] (the partial pressure of a neutral Ar gas is $(3–6)\times 10^{-4}$ torr and the electron temperature is $T_e=0.1$ eV), the electron mean free path with respect to electron-neutral collisions is on the order of 10^4 cm, which is much larger than the length of the device (90 cm) and its diameter (40 cm) [14]. Consequently, under the experimental conditions of Refs. [5–7], the ionization source term in the evolutionary equation for the ion density should be independent on the electron density. In other cases, in laboratory and space complex plasmas it is necessary to check whether the source is due to conventional electron impact ionization of neutrals (a traditional approach when describing dusty plasmas—see, e.g., Refs. [15,16]) or not. If this is so, the ionization rate is proportional to the electron density. The ion momentum loss is due to collisions with microparticles and is determined by the ion momentum-transfer frequency ν_{id}.

In order to calculate the recombination and momentum-transfer frequencies, one has to integrate the corresponding cross sections over the ion distribution function. The magnitude of the ion drift velocity in DIA solitons can vary in a rather wide range—it can be of the order of the ion-acoustic velocity (i.e., much higher than the ion thermal velocity $v_{T_i}=\sqrt{T_i/m_i}$). Therefore, "true" ion (shifted Maxwellian) distribution, $f_i(\mathbf{v})=(2\pi v_{T_i}^2)^{-3/2}\exp[-(\mathbf{v}-\mathbf{u})^2/2v_{T_i}^2]$ should be used. The recombination frequency is given by $\nu_{\rm r}=n_d\int v\,\sigma_{\rm c}(v) f_i(\mathbf{v})d\mathbf{v}$, with $\sigma_{\rm c}(v)$ the OML expression for the collection (absorption) cross section [1,12,13]. The momentum-transfer frequency consists of two parts—collection and orbital: $\nu_{id}=\nu_{id}^{\rm coll}+\nu_{id}^{\rm orb}$ [17]. Absorbed ions lose entire momentum on a particle and, therefore, the momentum-transfer cross section due to collection is $\sigma_{\rm c}(v)$. The orbital cross section corresponds to the elastic ion scattering in the particle field, and we use the Coulomb scattering cross section $\sigma_{\rm s}(v)$ with modified Coulomb logarithm [13]. Thus, the momentum-transfer frequency is $\nu_{id}=n_d\int(\mathbf{u}\cdot\mathbf{v}/u^2)v[\sigma_{\rm c}(v)+\sigma_{\rm s}(v)]f_i(\mathbf{v})d\mathbf{v}$. After the integration, we get the frequencies $\nu_{\rm r}$, $\nu_{id}^{\rm coll}$, and $\nu_{id}^{\rm orb}$, which are functions of Z_d and u:

$$\nu_{\rm r}=\sqrt{2\pi}a^2 v_{T_i} n_d \tilde{u}^{-1}[\sqrt{\pi/2}\,\mathrm{erf}(\tilde{u}/\sqrt{2})(1+2\tau z+\tilde{u}^2)+\tilde{u}\,\exp(-\tilde{u}^2/2)],$$

$$\nu_{id}^{\rm coll}=\sqrt{2\pi}a^2 v_{T_i} n_d \tilde{u}^{-2}\{\sqrt{\pi/2}\,\mathrm{erf}(\tilde{u}/\sqrt{2})\tilde{u}[1+\tilde{u}^2+(1-\tilde{u}^{-2})(1+2\tau z)]+(1+2\tau z+\tilde{u}^2)\exp(-\tilde{u}^2/2)\},$$

$$\nu_{id}^{\rm orb}=\sqrt{2\pi}a^2 v_{T_i} n_d (2\tau z)^2\Lambda(\tilde{u})\tilde{u}^{-3}[\sqrt{\pi/2}\,\mathrm{erf}(\tilde{u}/\sqrt{2})-\tilde{u}\exp(-\tilde{u}^2/2)],$$

where $\tilde{u}=u/v_{T_i}$ is the drift velocity normalized to the ion thermal velocity, $\tau=T_e/T_i$ is the electron-to-ion temperature ratio, and $z=Z_d e^2/aT_e$ is the surface potential of a microparticle in units of T_e/e. Note, that the expression for the ion absorption current which corresponds to the obtained $\nu_{\rm r}$, is well known (see, e.g., Ref. [18]). Also, $\nu_{id}^{\rm orb}$ is determined by well-known expression for the momentum-transfer frequency, which is used in ordinary electron-ion plasma [19]. The only difference is that we use the modified formula for the Coulomb logarithm derived in Ref. [13]: $\Lambda(\tilde{u})\simeq\ln[(1+\beta)/(a/\lambda_{\rm D}+\beta)]$, with $\beta(\tilde{u})=z\tau(a/\lambda_{\rm D})(1+\tilde{u}^2)^{-1}$ and the effective screening length is defined as $\lambda_{\rm D}^{-2}=\lambda_{{\rm D}i}^{-2}(1+\tilde{u}^2)^{-1}+\lambda_{\rm De}^{-2}$ (where $\lambda_{{\rm D}e,i}=\sqrt{T_{e,i}/4\pi e^2 n_{e,i}}$ is the electron or ion Debye length). This formula for $\Lambda(\tilde{u})$ differs from the usual definition of the Coulomb logarithm, due to a much larger range of the ion-particle interaction (larger Coulomb radius). However, at $\beta\ll 1$ it reduces to a usual expression [17].

The kinetic equation for the microparticle charge is determined by the OML model,

$$\partial Z_d/\partial t=J_e-J_i, \qquad (3)$$

where the electron and ion fluxes on the particle surface are

$$J_e=2\sqrt{2\pi}a^2 v_{T_e} n_e\,\exp(-z),$$

$$J_i=(n_i/n_d)\nu_{\rm r}.$$

Equations (1)–(3) are closed by the Poisson equation,

$$\partial^2\varphi/\partial x^2=4\pi e(n_e+Z_d n_d-n_i). \qquad (4)$$

In the absence of perturbations, the quasineutrality condition $n_{i0}=n_{e0}+Z_{d0}n_d$ holds.

We emphasize here that for the description of DIA solitons we use the model [3] that enabled us to describe successfully the laboratory experiments [5,6] on DIA shocks. Another approach for the description of nonlinear perturbations in complex plasmas invokes viscosity in dusty motion (see, e.g., Ref. [6]). However, in a classical approach to describing complex plasmas (see, e.g., Ref. [1]) by Eq. (3) for microparticle charging, it is impossible to derive the general hydrodynamic equation that describes the evolution of the ion momentum and contains the viscosity term in a conventional hydrodynamic form.

III. WEAKLY DISSIPATIVE SOLITONS

DIA solitons can be accompanied by either positive or negative electrostatic potential φ [8]. The positive φ is the potential well for electrons. The commonly used assumption in this case is that the electrons are not trapped in the potential well. However, this assumption is violated when the following inequality is valid [20]:

$$t_{\mathrm{sol}} \gtrsim L_{\mathrm{sol}}/v_{T_e}, \tag{5}$$

where t_{sol} is the characteristic time of the soliton formation and L_{sol} is the soliton width. The magnitude of t_{sol} is of the order of a few ω_{pi}^{-1} (where $\omega_{pe,i}=\sqrt{4\pi e^2 n_{e,i}/m_{e,i}}$ is the electron or ion plasma frequency), the spatial scale L_{sol} is about several $\lambda_{\mathrm{D}e}$. Thus, $L_{\mathrm{sol}}/v_{T_e}\sim\omega_{pe}^{-1}$, and therefore inequality (5) normally holds. In this case, distribution of electrons is modified due to their *adiabatic* trapping [20] and is described by the Gurevich formula:

$$\frac{n_e}{n_{e0}}=\exp\left(\frac{e\varphi}{T_e}\right)\mathrm{erfc}\left(\sqrt{\frac{e\varphi}{T_e}}\right)+\frac{2}{\sqrt{\pi}}\sqrt{\frac{e\varphi}{T_e}}, \tag{6}$$

where $\mathrm{erfc}(\zeta)\equiv 1-\mathrm{erf}(\zeta)$ is the complimentary error function. The first term in Eq. (6) corresponds to free electrons, while the trapped electrons are represented by the second term. Note that the Gurevich distribution presumes trapped electrons to be collisionless. For practical purposes, this means that the average time between electron-neutral collisions should be longer than the time during which the soliton exists in the experimental setup ($\sim L_{\mathrm{set}}/U$, where L_{set} is the setup size and U is the soliton velocity). This requirement is well satisfied under the experimental conditions [5–7].

We start our analysis with the "conservative" case, neglecting ionization and dissipative terms on the right-hand side in Eqs. (1) and (2) and assuming $Z_d=\mathrm{const}$ in Eq. (4). Solitary solutions depend on the variable $x-Ut$. For rarefactive solitons (which can exist only in complex plasmas, [8]) φ is negative, which corresponds to a potential heel for electrons. Therefore, electrons obey the Boltzmann distribution. This case was studied in detail in Ref. [8]. For the case of compressive solitons, electrons have the Gurevich distribution [Eq. (6)]. From Eqs. (1), (2), and (4), we derive the following equation for the electrostatic potential:

$$\partial^2\varphi/\partial x^2=-\partial V/\partial\varphi, \tag{7}$$

where the Sagdeev potential $V(\varphi)$ is given by

$$V(\varphi)=1-\exp(\varphi)\mathrm{erfc}(\sqrt{\varphi})-\frac{2\sqrt{\varphi}}{\sqrt{\pi}}-P\varphi-\frac{4\varphi^{3/2}}{3\sqrt{\pi}} + M(1+P)(M-\sqrt{M^2-2\varphi}). \tag{8}$$

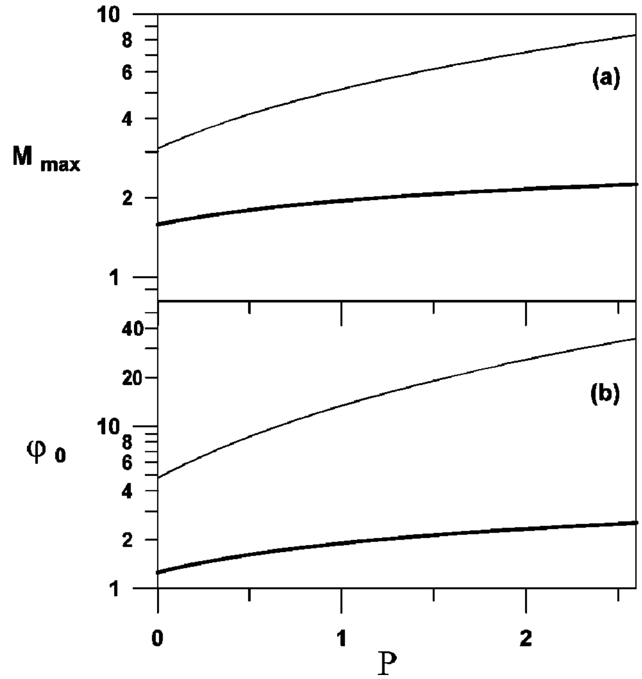

FIG. 1. Maximum value of Mach number $M_{\max}$ (a) and the maximum amplitude of the dimensionless electrostatic potential in the soliton φ_0 (b) versus the Havnes parameter P for compressive solitons with Gurevich electrons [Eq. (6), thin lines] and with Boltzmann electrons (bold lines).

Here we use the following normalization: $e\varphi/T_e\to\varphi$ and $x/\lambda_{De}\to x$, the Mach number is determined as $M=U/c_s$, with $c_s=\sqrt{T_e/m_i}$ being the ion acoustic velocity. The so-called "Havnes parameter," $P=Z_d n_d/n_e$, is a measure of the volume particle charge. Figure 1 shows that the maximum soliton amplitude [maximum of possible nonzero roots of Eq. (8)] for the Gurevich electron distribution is much higher than that for the Boltzmann distribution. The range of possible Mach numbers is much wider for the "Gurevich" soliton as well. Indeed, for the "Boltzmann" soliton the range of possible Mach numbers is rather narrow [8], e.g., for $P=2$ we have $2.16\geqslant M>1.73$. The analytical expression for the range of possible Mach numbers of the Gurevich soliton valid for $P>1$ is

$$\tfrac{9}{2}\pi(1+\tfrac{1}{2}P)\gtrsim M^2>1+P. \tag{9}$$

For $P=2$, Eq. (9) gives the range $7.5\gtrsim M>1.73$. This demonstrates the principal possibility to study experimentally the role of trapped electrons in the soliton evolution.

The time scale t_{diss}, which characterizes dissipation, is determined by the processes of microparticle charge variations and ion-particle collisions. It can be defined as $t_{\mathrm{diss}}\sim\min\{\nu_{\mathrm{r}}^{-1},\nu_{id}^{-1}\}$. If the characteristic time scale for dynamical processes is much shorter than t_{diss}, one can introduce a weakly dissipative system. For the DIA soliton the dynamical time scale is $\sim\omega_{pi}^{-1}$ and the dissipative time scale is $\sim P^{-1}(\lambda_{\mathrm{D}i}/a)(T_i/T_e)\omega_{pi}^{-1}$. Thus, we can refer to a weakly dissipative soliton when $\omega_{pi}t_{\mathrm{diss}}\gg 1$ (e.g., sufficiently small microparticles or/and low particle density).

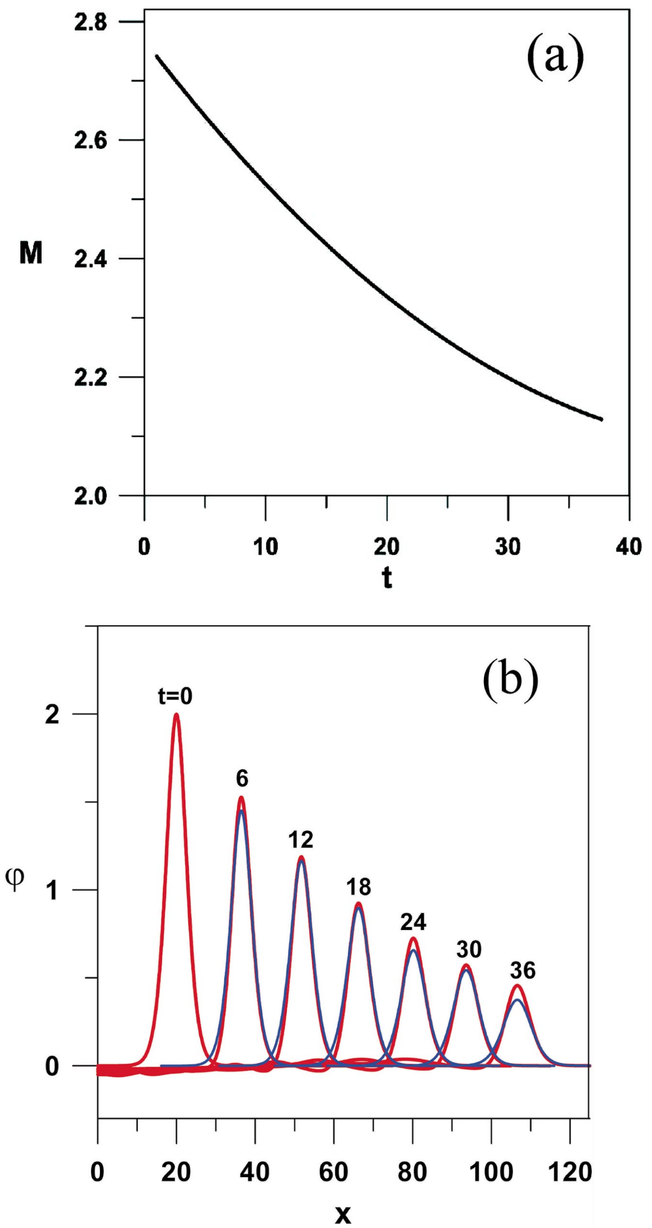

FIG. 2. (Color) Temporal evolution of the compressive solitonlike perturbation. Initial perturbation has the form of conservative soliton containing trapped electrons. It is given by the solution of Eqs. (7) and (8) for the Mach number $M=2.8$ and the Havnes parameter $P=2$. (a) Mach number of the perturbation versus dimensionless time t (time is normalized to $\sqrt{1+P}\,\omega_{pi}^{-1}$). (b) Dimensionless electrostatic potential φ versus dimensionless coordinate x. The red lines represent the numerically calculated profiles of the evolving perturbation at $t_i=0,6,12,18,24,30,36$. The blue lines show the conservative soliton solutions corresponding to $M(t_i)$. Plasma parameters: argon ion density $n_{i0}=3\times10^8$ cm^{-3}, particle radius $a=4.4$ μm, electron and ion temperatures $T_e=1.5$ eV and $T_i=0.1$ eV.

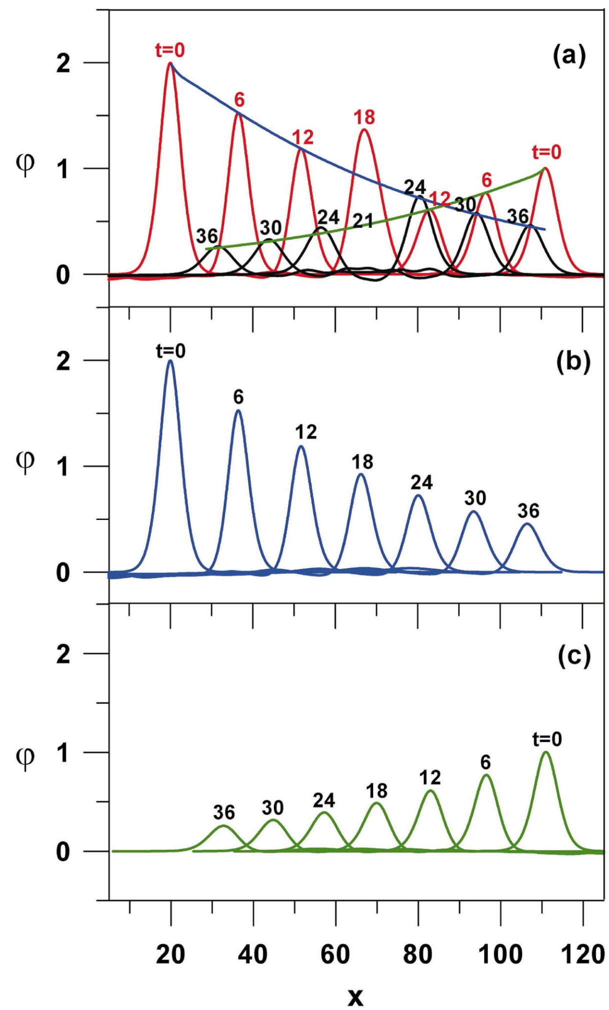

FIG. 3. (Color) Interaction of two weakly dissipative compressive solitons. In (a) we show the interaction of the solitons whose temporal evolution (in the absence of interaction) is given in (b) and (c). The red and black lines correspond to the soliton profiles dimensionless electrostatic potential φ versus dimensionless coordinate x before and after the interaction, respectively, for different values of dimensionless time $t_i=0,6,12,18,21,24,30,36$ (time is normalized to $\sqrt{1+P}\,\omega_{pi}^{-1}$). Initially, both perturbations have the form of "conservative" soliton solution [Eqs. (7) and (8)]. The initial soliton amplitude $\varphi_0=2$ (b) and 1 (c), the Mach number $M=2.8$ (b) and 2.44 (c), the Havnes parameter $P=2$. The blue and green curves in (a) represent the amplitude evolution of individual solitons shown in (b) and (c), respectively. Plasma parameters: argon ion density $n_{i0}=3\times10^8$ cm^{-3}, particle radius $a=4.4$ μm, electron and ion temperatures $T_e=1.5$ eV and $T_i=0.1$ eV.

We investigated the evolution of the solitonlike perturbations in complex plasmas, taking into account the dissipation processes. The evolution of weakly dissipative compressive perturbations containing the trapped electrons occurs in the following manner: The perturbation is damped and its speed (the Mach number M) decreases monotonically with time, as shown in Fig. 2(a). However, Fig. 2(b) demonstrates that the form of the evolving perturbation at any moment is given by the "conservative" soliton solution [Eqs. (7) and (8)] for the corresponding Mach number. Investigation of the interaction of two weakly dissipative solitons containing the trapped electrons (see Fig. 3) shows that, after the interaction, each perturbation keeps the form of the soliton propagating individually from the beginning. That property is inherent in all solitons.

IV. CONCLUSIONS AND SUMMARY

Thus, we can conclude that ''weakly dissipative'' solitons can exist. We emphasize, however, that some specific conditions for their excitation should be fulfilled. First of all, the velocity of the initial perturbation should satisfy the range of Mach numbers of the corresponding soliton, e.g., Eq. (9) for the ''Gurevich'' soliton in the $P>1$ case. We have performed numerical investigations of different initial solitonlike perturbations of both the ''Boltzmann'' and ''Gurevich'' type. In all cases, Boltzmann solitonlike initial perturbations were converted to shocklike structures. We have shown the existence of the damped solitons only for the Gurevich electron distribution. We explain this by the fact that the speed of the perturbations decreases monotonically with time. If the Mach number range is rather narrow (that takes place always for ''Boltzmann'' solitons), then during a short period of time the speed of perturbation becomes less than the minimum soliton speed, and the soliton disappears. Consequently, for the existence of the damped solitons the perturbation should have the initial form, so that it would allow the presence of both free and trapped electrons. Otherwise, there is a possibility of an appearance of DIA shocks in complex plasmas. For example, the initial perturbation in the form of a motionless region with a constant enhanced ion density (similar to that excited in the experiments [5] on DIA shocks) does not contain the trapped electrons and may not evolve in the soliton form.

In summary, our calculations show a possibility for the so-called ''weakly-dissipative'' DIA compressive solitons to exist. Their form is given by the ''conservative'' soliton solution at the appropriate Mach number. Interacting solitons conserve their form. However, in contrast to the usual solitons, the total energy and the total momentum decrease monotonically with time. The role of trapped electrons in such solitons is significant, hence the study of observable properties can be used as a diagnostic tool to investigate microscopic properties of the electrons.

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